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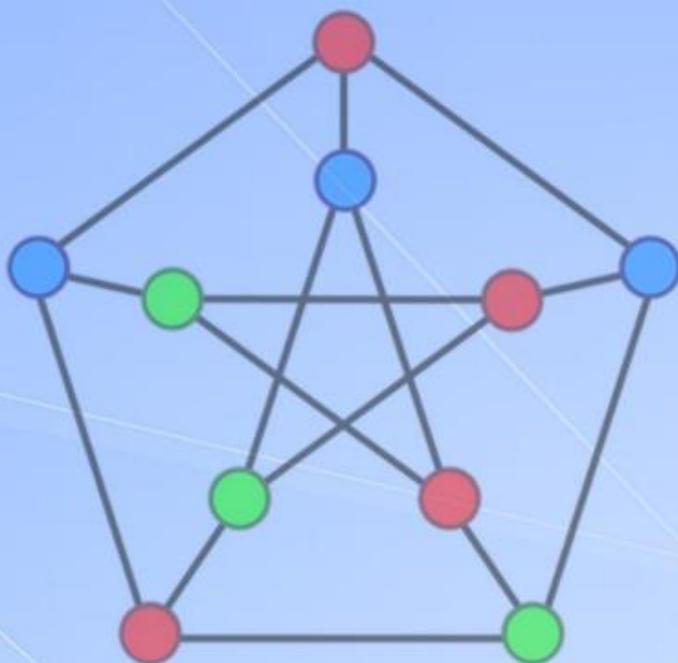
**The 6th Xi'an International Workshop on  
Graph Theory and Combinatorics**

**第六届西安国际图论与组合数学研讨会**

**Program**

**程序册**

**Northwestern Polytechnical University**



**June 23-27, 2022**

**Xi'an, Shaanxi, China**

# The 6th Xi'an International Workshop on Graph Theory and Combinatorics

Northwestern Polytechnical University

Xi'an, Shaanxi, P.R. China

June 23-June 27, 2022

## Location

ZOOM Meeting ID: 891 1441 8590, Passcode: xian

## Program Committee

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**Members:** Hajo Broersma, Gyula O.H. Katona, Hao Li, Xueliang Li, You Lu, Miklós Simonovits, Ligong Wang, Shenggui Zhang

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Email: bai@nwpu.edu.cn

# The 6th Xi'an International Workshop on Graph Theory and Combinatorics

## Program

Location: ZOOM Meeting 891 1441 8590		Passcode: xian	
June 23, Thursday			
13:45-14:00	Opening Ceremony		Chair: Shenggui Zhang
Beijing Time	Chair	Speaker	Title
14:00-14:45	Baogang Xu	Miklós Simonovits	The influence of extremal graph theory on discrete mathematics
14:45-15:30		Xueliang Li	Some new results on circular flows and group connectivity of graphs
15:30-15:45	Break		
15:45-16:15	Jiuqiang Liu	Xianan Jin	A proof of the well-definedness of the interior and exterior polynomials
16:15-16:45		Edwin R. van Dam	Pseudo-geometric strongly regular graphs with a regular point
16:45-17:15		Suijie Wang	Bijections on hyperplane arrangements
17:15-19:00	Break		
19:00-19:45	Jiaao Li	Linyuan Lu	Anti-Ramsey number of disjoint rainbow bases in all matroids
19:45-20:15		Qizhong Lin	Fan-complete Ramsey numbers
20:15-20:45		Xihe Li	Problems and results on the Erdős-Gyárfás problem
20:45-21:00	Break		
21:00-21:45	Hehui Wu	Zhongshan Li	Signed graphs with maximum nullity at most two
21:45-22:30		Hong-Jian Lai	Supereulerian matroids
June 24, Friday			
14:00-14:45	Ligong Wang	Yuejian Peng	The bipartite Ramsey number of cycles
14:45-15:30		Liyang Kang	Spectral extremal graphs for intersecting cliques
15:30-15:45	Break		
15:45-16:15	Yaping Mao	Boram Park	Odd coloring of graphs
16:15-16:45		Min Chen	A sufficient condition for planar graphs to be $(3,1)^*$ -choosable
16:45-17:15		Shinya Fujita	New classification of graphs in view of the domination number of central graphs
17:15-19:00	Break		
19:00-19:45	Yujun Yang	Gregory Gutin	Lower bounds for weighted max cut
19:45-20:30		Liming Xiong	How does an induced Hourglass effect a graph to have Hamiltonian properties?
20:30-20:45	Break		

Beijing Time	Chair	Speaker	Title
20:45-21:15	Xiaogang Liu	Jianfeng Wang	On the eccentricity matrix of finite graphs
21:15-21:45		Huiqiu Lin	Recent results on the spectral extremal graph theory
21:45-22:15		Fenjin Liu	Some constructions of singularly cospectral graphs
<b>June 25, Saturday</b>			
14:00-14:45	Libo Yang	Gyula O.H. Katona	Some new Turán type theorems
14:45-15:30		Dragan Stevanović	On Hosoya's dormants and sprouts
15:30-15:45	Break		
15:45-16:30	Jinxin Zhou	Xiande Zhang	Bollobás-type theorems for hemi-bundled two families
16:30-17:15		Baoxuan Zhu	Some properties from real-rooted polynomials in combinatorics
17:15-17:45		Xiaoyan Zhang	New routing problem in FPGA chip design
17:45-19:00	Break		
19:00-19:30	Weihua Yang	Bo Ning	A complete solution to the Cvetković–Rowlinson conjecture
19:30-20:00		Shenwei Huang	Some results on $k$ -critical $P_5$ -free graphs
20:00-20:30		Xia Liu	Forbidden pairs of disconnected graphs for supereulerianity of connected graphs
20:30-20:45	Break		
20:45-21:15	Erfang Shan	Yiqiao Wang	Star edge coloring of 1-planar graphs
21:15-21:45		Ran Gu	$\ell$ -(edge)-connectivity of random graphs
21:45-22:15		Jun Gao	Tight bounds on a conjecture of Gallai
<b>June 26, Sunday</b>			
14:00-14:45	Jin Yan	Douglas B. West	Cycles in color-critical graphs
14:45-15:30		Hao Li	A generalization of Hamilton-connection
15:30-15:45	Break		
15:45-16:15	Zhang Zhang	Adam Kabela	Forbidden induced pairs and additional constraints for perfectness and $\omega$ -colourability of graphs
16:15-16:45		Xing Peng	Anti-Ramsey number of hypergraph matchings
16:45-17:15		Tianchi Yang	Counting critical subgraphs in $k$ -critical graphs
<b>17:15-17:30</b>	<b>Closing Ceremony Chair: Shenggui Zhang</b>		

# Time Differences from UTC to capital cities of countries of the workshop's participants

Capital City,Country	First Day of Workshop	Local Time
 <b>Beijing, China</b> CST (UTC +8)	Thu, 23 Jun 2022	<b>14:00</b>
 <b>Prague, Czechia</b> CEST (UTC +2)	Thu, 23 Jun 2022	<b>08:00</b>
 <b>Paris, France</b> CEST (UTC +2)	Thu, 23 Jun 2022	<b>08:00</b>
 <b>Budapest, Hungary</b> CEST (UTC +2)	Thu, 23 Jun 2022	<b>08:00</b>
 <b>Tokyo, Japan</b> JST (UTC +9)	Thu, 23 Jun 2022	<b>15:00</b>
 <b>Amsterdam, Netherlands</b> CEST (UTC +2)	Thu, 23 Jun 2022	<b>08:00</b>
 <b>Belgrade, Serbia</b> CEST (UTC +2)	Thu, 23 Jun 2022	<b>08:00</b>
 <b>Seoul, South Korea</b> KST (UTC +9)	Thu, 23 Jun 2022	<b>15:00</b>
 <b>Washington DC, DC, USA</b> EDT (UTC -4)	Thu, 23 Jun 2022	<b>02:00</b>
 <b>London, United Kingdom</b> BST (UTC +1)	Thu, 23 Jun 2022	<b>07:00</b>

Observing [Daylight Saving Time](#)

# Abstract

## 1 A sufficient condition for planar graphs to be $(3, 1)^*$ -choosable

Min Chen

Zhejiang Normal University, China

chenmin@zjnu.cn

Let  $d_1, d_2, \dots, d_k$  be  $k$  non-negative integers. A graph  $G$  is  $(d_1, \dots, d_k)$ -colorable if there exists a mapping  $\pi : V(G) \rightarrow \{1, \dots, k\}$  such that the subgraph  $G[i]$  induced by color  $i$  has maximum degree at most  $d_i$  for  $i = 1, 2, \dots, k$ . Especially, if  $d_1 = d_2 = \dots = d_k$ , then such kind of coloring is called a  $(d, d, \dots, d)$ -coloring.

An  $(L, d)^*$ -coloring is a mapping  $\pi$  that assigns a color  $\pi(v) \in L(v)$  to each vertex  $v \in V(G)$  so that at most  $d$  neighbors of  $v$  receive color  $\pi(v)$ . A graph  $G$  is said to be  $(k, d)^*$ -choosable if it admits an  $(L, d)^*$ -coloring for every list assignment  $L$  with  $|L(v)| \geq k$  for all  $v \in V(G)$ .

In this talk, we will show the proof that every planar graph without adjacent  $i$ -cycles for each  $i \in \{3, 4, 5\}$  is  $(3, 1)^*$ -choosable. This is joint work with Jufeng Zhang and Yiqiao Wang.

## 2 New classification of graphs in view of the domination number of central graphs

Shinya Fujita

Yokohama City University, Japan

shinya.fujita.ph.d@gmail.com

For a graph  $G$ , the central graph  $C(G)$  is the graph constructed from  $G$  by subdividing each edge of  $G$  with one vertex and also by adding an edge to every pair of non-adjacent vertices in  $G$ . Also for a graph  $G$ , let  $\gamma(G)$  and  $\tau(G)$  be the domination number of  $G$  and the minimum cardinality of a vertex cover of  $G$ , respectively. In this talk, we give a new classification of graphs concerning the domination number of central graphs and minimum vertex covers of graphs. Namely, we show that any graph  $G$  with at least three vertices can be classified into one of the two classes of graphs with  $\gamma(C(G)) = \tau(G)$  and  $\gamma(C(G)) = \tau(G) + 1$ , respectively, together with some special properties concerning a vertex cover of  $G$ . We also give some new results on the domination number of central graphs. This is joint work with Farshad Kazemnejad and Behnaz Pahlavansay.

## 3 Tight bounds on a conjecture of Gallai

Jun Gao

University of Science and Technology of China, China

gj0211@mail.ustc.edu.cn

We prove that for  $n > k \geq 3$ , if  $G$  is an  $n$ -vertex graph with chromatic number  $k$  but any its proper subgraph has smaller chromatic number, then  $G$  contains at most  $n - k + 3$  copies of cliques of size  $k - 1$ . This answers a problem of Abbott and Zhou and provides a tight bound on a conjecture of Gallai.

## 4 $\ell$ -(edge)-connectivity of random graphs

Ran Gu

Hohai University, China

guran323@163.com

For an integer  $\ell \geq 2$ , the  $\ell$ -connectivity  $\kappa_\ell(G)$  of a graph  $G$  is defined to be the minimum number of vertices of  $G$  whose removal produces a disconnected graph with at least  $\ell$  components or a graph with fewer than  $\ell$  vertices. The  $\ell$ -edge-connectivity  $\lambda_\ell(G)$  of a graph  $G$  is the minimum number of edges whose removal leaves a graph with at least  $\ell$  components if  $|V(G)| \geq \ell$ , and  $\lambda_\ell(G) = |E(G)|$  if  $|V(G)| < \ell$ . Given integers  $k \geq 0$  and  $\ell \geq 2$ , we investigate  $\kappa_\ell(G(n, p))$  and  $\lambda_\ell(G(n, p))$  when  $np \leq \log n + k \log \log n$ . Furthermore, our arguments can be used to show that in the random graph process, the hitting times of minimum degree at least  $k$  and of  $\ell$ -connectivity (or  $\ell$ -edge-connectivity) at least  $k(\ell - 1)$  coincide with high probability. These results generalize the work of Bollobás and Thomason on classical connectivity.

## 5 Lower bounds for Weighted Max Cut

Gregory Gutin

Royal Holloway, University of London, UK

G.Gutin@rhul.ac.uk

While there have been many results on lower bounds for Max Cut in unweighted graphs, the only lower bound for non-integer weights is that by Poljak and Turzik (1986). We'll discuss an extensive study of lower bounds for Max Cut in weighted graphs. We'll introduce a new approach for obtaining lower bounds for Weighted Max Cut. Using it, Probabilistic Method, Vizing's chromatic index theorem, and other tools, we can obtain several lower bounds for arbitrary weighted graphs, weighted graphs of bounded girth and triangle-free weighted graphs. We'll pose conjectures and open questions. This talk is based on a joint paper with Anders Yeo.

## 6 Some results on $k$ -critical $P_5$ -free graphs

Shenwei Huang

Nankai University, China

shenweihuang@nankai.edu.cn

A graph  $G$  is  $k$ -vertex-critical if  $G$  has chromatic number  $k$  but every proper induced subgraph of  $G$  has chromatic number less than  $k$ . The study of  $k$ -vertex-critical graphs for graph classes is an important topic in algorithmic graph theory because if the number of such graphs that are in a given hereditary graph class is finite, then there is a polynomial-time algorithm to decide if a graph in the class is  $(k - 1)$ -colorable.

In this talk, we prove that for every fixed integer  $k \geq 1$ , there are only finitely many  $k$ -vertex-critical  $(P_5, \text{gem})$ -free graphs and  $(P_5, \overline{P_3 + P_2})$ -free graphs. To prove the results we use a known structure theorem for  $(P_5, \text{gem})$ -free graphs combined with properties of  $k$ -vertex-critical graphs. Moreover, we characterize all  $k$ -vertex-critical  $(P_5, \text{gem})$ -free graphs and  $(P_5, \overline{P_3 + P_2})$ -free graphs for  $k \in \{4, 5\}$  using a computer generation algorithm.

## 7 A proof of the well-definedness of the interior and exterior polynomials

Xianan Jin  
Xiamen University, China  
xajin@xmu.edu.cn

The interior polynomial and the exterior polynomial, introduced in [1], are generalizations of valuations on  $(x, 1)$  and  $(1, y)$  of the Tutte polynomial  $T_G(x, y)$  of graphs to hypergraphs, respectively. The top of the HOMFLY polynomial of a special alternating link coincides with the interior polynomial of the pair of hypergraphs induced by the Seifert graph of the link. In this talk, we will give a detailed account of definitions of these two polynomials, their relations with the Tutte polynomial in graph theory and the HOMFLY polynomial in knot theory. The two polynomials are defined under a fixed ordering of hyperedges, and are proved to be independent of the ordering using techniques of polytopes in [1]. Similar to the Tutte's proof of his polynomial, we then provide a direct and elementary proof for the well-definedness of the interior and exterior polynomials of hypergraphs. See [2] for details.

- [1] T. Kalman, A version of Tutte's polynomial for hypergraphs, *Adv. Math.* 244 (2013) 823-873.
- [2] X. Guan, X. Jin, T. Ma, A direct and elementary proof of the well-definedness of the interior and exterior polynomials of hypergraphs, arXiv:2201.12496[math.CO].

## 8 Forbidden induced pairs and additional constraints for perfectness and $\omega$ -colourability of graphs

Adam Kabela  
University of West Bohemia, Czech Republic  
kabela@kma.zcu.cz

We characterise the pairs of graphs  $\{X, Y\}$  such that all  $\{X, Y\}$ -free graphs (distinct from  $C_5$ ) are perfect. Similarly, we characterise pairs  $\{X, Y\}$  such that all  $\{X, Y\}$ -free graphs (distinct from  $C_5$ ) are  $\omega$ -colourable (that is, their chromatic number is equal to their clique number). More generally, we show characterizations of pairs  $\{X, Y\}$  for perfectness and  $\omega$ -colourability of all connected  $\{X, Y\}$ -free graphs which are of independence at least 3, distinct from an odd cycle, and of order at least  $n_0$ , and similar characterisations subject to each subset of these additional constraints.

The considered classes of graphs are non-hereditary and the characterisations for perfectness and  $\omega$ -colourability are different. We discuss the additional constraints and relate the present characterisations to known results on forbidden induced pairs for  $\chi$ -boundedness and for deciding  $k$ -colourability in polynomial time, and we mention several open problems from these areas. We use the fact that pairs of graphs can be represented by a graph and provide figures depicting the collections of the forbidden pairs characterised.

The talk is based on joint work with Maria Chudnovsky, Binlong Li and Petr Vrána and previous joint work with Christoph Brause, Přemysl Holub, Zdeněk Ryjáček and Ingo Schiermeyer.

## 9 Spectral extremal graphs for intersecting cliques

Liyang Kang  
Shanghai University, China  
lykang@shu.edu.cn

The  $(k, r)$ -fan is the graph consisting of  $k$  copies of the complete graph  $K_r$  which intersect in a single vertex, and is denoted by  $F_{k,r}$ . Erdős, Füredi, Gould and Gunderson [J. Combin. Theory Ser. B 64 (1995) 89–100] determined “the maximum number of edges in an  $n$ -vertex graph that does not contain  $F_{\{k, 3\}}$  as a subgraph. Furthermore, Chen, Gould, Pfender and Wei [J. Combin. Theory Ser. B 89 (2003) 159–171] proved the analogous result on  $F_{k,r}$  for the general case  $r \geq 3$ . In this paper, we show that for sufficiently large  $n$ , the graphs of order  $n$  that contain no copy of  $F_{k,r}$  and attain the maximum spectral radius are also edge-extremal. That is, such graphs must have  $ex(n, F_{k,r})$  edges.

## 10 Some new Turán type theorems

Gyula O.H. Katona  
MTA Rényi Institute, Hungary  
ohkatona@renyi.hu

Mantel proved in 1907 that if a graph  $G$  with  $n$  vertices contains no triangle then the number of its edges is at most  $\lfloor \frac{n^2}{4} \rfloor$ . The optimal graph is a complete bipartite graph with equal parts. In 1941 Turán found a generalization: if  $G$  contains no copy of the complete graph  $K_{k+1}$  then the number of its edges is not more than the complete  $k$ -partite graph with  $k$  equal parts has. In general a Turán type problem asks for the maximum number of edges in a graph  $G$  with  $n$  vertices containing no copy of a “small” given graph  $H$ . This maximum is denoted by  $ex(n, H)$ . Mantel’s and Turán’s theorems determined  $ex(n, K_3)$  and  $ex(n, K_{k+1})$ , respectively. Rademacher observed that if  $G$  has  $ex(n, K_3) + 1$  edges then there are suddenly at least  $\lfloor \frac{n}{2} \rfloor$  triangles, not only one. We prove a strengthening of this statement, namely if the number of edges is  $ex(n, K_3) + 1$  and no vertex pins down all triangles then their number is at least  $n - 2$ .

Let  $P_k$  denote a path of  $k$  vertices. Erdős and Gallai determined  $ex(n, P_k)$ . Our new result gives the solution of the combined problem: we found  $ex(n, \{K_m, P_k\})$  for large  $n$ .

The results are jointly achieved with Chuanqi Xiao.

## 11 Supereulerian matroids

Hong-Jian Lai  
West Virginia University, USA  
Hong-jian.Lai@mail.wvu.edu

The supereulerian graph problem was initiated by Boesch, Suffey and Tindel in 1977 and has been intensively studied. In the Math Review of Welsh’s book “Matroid Theory”, Tutte indicated that a trend of matroid theory would be to extend known results in graph theory to matroids and to argue without mentioning vertices. We will introduce some of the recent attempts to extend supereulerian graph results to matroids, and discuss future research problems in this direction.

## 12 A generalization of Hamilton-connection

Hao Li  
Université Paris-Saclay, France  
li@lri.fr

To be decided.

## 13 Problems and results on the Erdős-Gyárfás problem

Xihe Li  
Northwestern Polytechnical University, China  
lxhdhr@163.com

For two graphs  $G, H$  and an integer  $q$  with  $2 \leq q \leq |E(H)|$ , an  $(H, q)$ -coloring of  $G$  is an edge-coloring of  $G$  such that every copy of  $H$  in  $G$  receives at least  $q$  distinct colors. Let  $r(G, H, q)$  be the minimum number of colors that are needed for  $G$  to have an  $(H, q)$ -coloring. In particular,  $r(K_n, K_p, q)$  is usually written as  $f(n, p, q)$  and known as the Erdős-Gyárfás function;  $r(K_{n,n}, K_{s,t}, q)$  is known as the bipartite Erdős-Gyárfás function. This problem was introduced by Erdős and Shelah about 45 years ago, but was studied systematically by Erdős and Gyárfás in 1997, and it can be viewed as a generalization of the classical Ramsey number.

In this talk, we first introduce our work on  $f(n, p, q)$  with respect to Gallai-colorings, where a Gallai-coloring is an edge-coloring of  $K_n$  without rainbow triangles. Namely, we consider the function  $g(n, p, q)$  that is the minimum number of colors needed for  $K_n$  to have a Gallai-coloring in which every copy of  $K_p$  receives at least  $q$  distinct colors. Secondly, we introduce our work on the asymptotic behavior of the bipartite Erdős-Gyárfás function  $r(K_{n,n}, K_{s,t}, q)$ , as well as the new developed bipartite color energy method. In addition, we give a short survey on known results and interesting open problems on the Erdős-Gyárfás problem and related topics, including a hypergraph version and some problems in discrete geometry and additive combinatorics.

Joint work with Hajo Broersma (University of Twente) and Ligong Wang (Northwestern Polytechnical University).

## 14 Some new results on circular flows and group connectivity of graphs

Xueliang Li  
Nankai University, China  
lxl@nankai.edu.cn

Tutte proposed integer flows when he studied the well-known 4-Color Problem in 1949. For further study of flow properties, Jaeger et al. (1992) introduced the concept of group connectivity as a nonhomogeneous analogue of integer flows, and Goddyn et al. (1998) proposed the definition of circular flows which extends the range of flows to arbitrary rational numbers. In this talk, we will discuss the existence of circular flow for regular Class  $I$  graphs and the equivalence of group connectivity on non-isomorphic groups with a same order. Joint work with Jiaao Li and Meiling Wang.

## 15 Signed graphs with maximum nullity at most two

Zhongshan Li  
Georgia State University, USA  
zli@gsu.edu

A signed graph is an ordered pair  $(G, \Sigma)$ , where  $G = (V, E)$  is a graph (in which parallel edges are permitted but loops are not), with  $V = \{1, 2, \dots, n\}$  and  $\Sigma \subseteq E(G)$ . The edges in  $\Sigma$  are called odd, and the edges in  $E(G) \setminus \Sigma$  are even. By  $\mathcal{S}(G, \Sigma)$  we denote the set of all real symmetric  $n \times n$  matrices  $A = [a_{ij}]$  with  $a_{ij} < 0$  if  $i$  and  $j$  are adjacent and all edges between  $i$  and  $j$  are even,  $a_{ij} > 0$  if  $i$  and  $j$  are adjacent and all edges between  $i$  and  $j$  are odd,  $a_{ij}$  is arbitrary if there exist both even and odd edges between  $i$  and  $j$ ,  $a_{ij} = 0$  if  $i$  and  $j$  are non-adjacent, and  $a_{ii}$  is arbitrary for all vertices  $i$ . The maximum nullity of the signed graph  $(G, \Sigma)$ , denoted  $M(G, \Sigma)$ , is the largest nullity of any matrix  $A \in \mathcal{S}(G, \Sigma)$ . A matrix  $A \in \mathcal{S}(G, \Sigma)$  has the Strong Arnold Property (SAP) with respect to  $(G, \Sigma)$  if  $X = [x_{ij}] = 0$  is the only matrix such that  $AX = 0$ , and  $x_{ij} = 0$  if  $i$  and  $j$  are adjacent or  $i = j$ . The stable maximum nullity of the signed graph  $(G, \Sigma)$ , denoted  $\xi(G, \Sigma)$ , is the largest nullity of any matrix  $A \in \mathcal{S}(G, \Sigma)$  with the SAP. Characterizations of signed graphs with maximum nullity at most 2 or with maximum stable nullity at most 2 are presented. A brief survey of a related parameter, the Colin de Verdière invariant of a simple graph is also given.

## 16 Recent results on the spectral extremal graph theory

Huiqiu Lin  
East China University of Science and Technology, China  
huiqiulin@ecust.edu.cn

In this talk, we will survey results on the spectral extremal graph theory. Very recently, some classic conjectures are solved, including the Nikiforov's even cycle conjecture and Tait's conjecture on  $K_{\ell s, t}$ -minor free graphs.

## 17 Fan-complete Ramsey numbers

Qizhong Lin  
Fuzhou University, China  
linqizhong@fzu.edu.cn

We consider Ramsey numbers  $r(G, H)$  with tight lower bounds, namely,

$$r(G, H) \geq (\chi(G) - 1)(|H| - 1) + 1$$

where  $\chi(G)$  denotes the chromatic number of  $G$  and  $|H|$  denotes the number of vertices in  $H$ . We say  $H$  is  $G$ -good if the equality holds.

In this paper, we prove that the fan-graph  $F_n = K_1 + nK_2$  is  $K_p$ -good if  $n \geq 27p^2$ , improving previous tower-type lower bounds for  $n$  due to Li and Rousseau (1996). The join graph  $G + H$  is defined by adding all edges between the disjoint vertex sets of  $G$  and  $H$ . Let  $nH$  denote the union graph of  $n$  disjoint copies of  $H$ . We show that  $K_1 + nH$  is  $K_p$ -good if  $n \geq 27\ell p$  where  $\ell = r(K_p, H)$ . We give a stronger lower bound inequality for Ramsey number  $r(G, K_1 + F)$  for the case of  $G = K_p(a_1, a_2, \dots, a_p)$ , the complete  $p$ -partite graph with  $a_1 = 1$  and  $a_i \leq a_{i+1}$ . In particular, using a stability-supersaturation lemma by Fox, He and Wigderson (2021), we show that for any fixed graph  $H$ ,

$$r(G, K_1 + nH) = \begin{cases} (p-1)(n|H| + a_2 - 1) + 1 & \text{if } n|H| + a_2 - 1 \text{ or } a_2 - 1 \text{ is even,} \\ (p-1)(n|H| + a_2 - 2) + 1 & \text{otherwise,} \end{cases}$$

where  $G = K_p(1, a_2, \dots, a_p)$  with  $a_i$ 's satisfying some mild conditions and  $n$  is sufficiently large. The special case of  $H = K_1$  gives an answer to Burr's question (1981) about the discrepancy of  $r(G, K_{1,n})$  from  $G$ -goodness. Joint work with Fan Chung.

## 18 Some constructions of singularly cospectral graphs

Fenjin Liu  
Chang'an University, China  
fenjinliu@163.com

Two graphs are singularly cospectral if they have same nonzero singular values with their multiplicity. In this talk, we summarize some equivalent conditions for singularly cospectral graphs. Furthermore, we give three methods to construct a pair of singularly cospectral graphs with same order, which includes the operation of join, corona and the  $m$ -shadow graph.

## 19 Forbidden pairs of disconnected graphs for supereulerianity of connected graphs

Xia Liu  
Northwestern Polytechnical University, China  
liuxia\_math@nwpu.edu.cn

Let  $\mathcal{H}$  be a set of graphs. A graph  $G$  is said to be  $\mathcal{H}$ -free if  $G$  does not contain  $H$  as an induced subgraph for all  $H \in \mathcal{H}$ . A graph is called supereulerian if it contains a spanning connected even subgraph. Lv and Xiong used the forbidden induced subgraphs condition to investigate the supereulerianity.

We characterize all pairs of graphs  $R, S$  (not necessary connected) such that every 2-connected or 2-edge-connected  $\{R, S\}$ -free graph (of sufficiently large order) is supereulerian. As a byproduct, we also characterize all minimal 2-connected non-supereulerian graphs.

This is joint work with Binlong Li and Shipeng Wang.

## 20 Anti-Ramsey number of disjoint rainbow bases in all matroids

Linyuan Lu  
University of South Carolina, USA  
lu@math.sc.edu

Consider a matroid  $M = (E, I)$  with its elements of the ground set  $E$  colored. A *rainbow basis* is a maximum independent set in which each element receives a different color. The *rank* of a subset  $S$  of  $E$ , denoted by  $r_M(S)$ , is the maximum size of an independent set in  $S$ . A *flat*  $F$  is a maximal set in  $M$  with a fixed rank. The *anti-Ramsey* number of  $t$  pairwise disjoint rainbow bases in  $M$ , denoted by  $ar(M, t)$ , is defined as the maximum number of colors  $m$  such that there exists an  $m$  coloring of the ground set  $E$  of  $M$  which contains no  $t$  pairwise disjoint rainbow bases. We determine  $ar(M, t)$  for all matroids of rank at least 2:  $ar(M, t) = |E|$  if there exists a flat  $F_0$  with  $|E| - |F_0| < t(r_M(E) - r_M(F_0))$ ; and  $ar(M, t) = \max_{F: r_M(F) \leq r_M(E) - 2} \{|F| + t(r_M(E) - r_M(F) - 1)\}$  otherwise. (Joint work with Andrew Meier.)

## 21 A Complete Solution to the Cvetković-Rowlinson Conjecture

Bo Ning  
Nankai University, China  
bo.ning@nankai.edu.cn

In 1990, Cvetković and Rowlinson [The largest eigenvalue of a graph: a survey, *Linear Multilinear Algebra* 28(1-2) (1990), 3-33] conjectured that among all outerplanar graphs on  $n$  vertices,  $K_1 \vee P_{n-1}$  attains the maximum spectral radius. In 2017, Tait and Tobin [Three conjectures in extremal spectral graph theory, *J. Combin. Theory, Ser. B* 126 (2017) 137-161] confirmed the conjecture for sufficiently large values of  $n$ . In this talk, we present a complete proof of this conjecture. (Joint work with Huiqiu Lin)

## 22 Odd coloring of graphs

Boram Park  
Ajou University, South Korea  
borampark@ajou.ac.kr

In this talk, we consider an odd coloring of a graph, which was introduced very recently, motivated by parity type colorings of graphs. A proper vertex coloring of graph  $G$  is said to be odd if for each non-isolated vertex  $x \in V(G)$  there exists a color  $c$  such that  $c$  is used an odd number of times in the neighborhood of  $x$ . The recent work on this topic will be presented, and the work is based on Eun-Kyung Cho, Ilkyoo Choi, and Hyemin Kwon.

## 23 Anti-Ramsey number of hypergraph matchings

Xing Peng  
Anhui University, China  
x2peng@ahu.edu.cn

An  $s$ -matching  $M_s$  in a hypergraph is a set of  $s$  pairwise disjoint edges. The anti-Ramsey number  $ar(n, k, M_s)$  of an  $s$ -matching is the smallest integer  $c$  such that each edge-coloring of the  $n$ -vertex  $k$ -uniform complete hypergraph with exactly  $c$  colors contains an  $s$ -matching with distinct colors. In 2013, Ozkahya and Young proposed a conjecture on the exact value of  $(n, k, M_s)$ . Frankl and Kupavskii verified this conjecture for small  $s$ . In this talk, I will present a recent result on this conjecture. This is joint work with Mingyang Guo and Hongliang Lu.

## 24 The bipartite Ramsey number of cycles

Yuejian Peng  
Hunan University, China  
ypeng1@hnu.edu.cn

Given bipartite graphs  $F$  and  $G$ , the bipartite Ramsey number  $br(F, G)$  is the minimum integer  $N$  such that any red-blue-edge-coloring of the complete bipartite graph  $K_{N, N}$  contains a red  $F$  or a blue  $G$ . There are considerable results on asymptotic values of bipartite Ramsey numbers of cycles. For exact values, Zhang-Sun determined  $br(C_4, C_{2n})$ , Zhang-Sun-Wu determined  $br(C_6, C_{2n})$ , and Gholami-Rowshan determined  $br(C_8, C_{2n})$ . We solve all remaining cases and give the exact values of  $br(C_{2n}, C_{2m})$  for all  $n \geq m \geq 5$ , this answers a question concerned by Bucić-Letzter-Sudakov, Gholami-Rowshan, Zhang-Sun, and Zhang-Sun-Wu. This is a joint work with Yan Zilong.

## 25 The influence of extremal graph theory on Discrete Mathematics

Miklós Simonovits  
Alfréd Rényi Math Institute, Hungary  
miki@renyi.hu

Roughly a year ago I gave a lecture with the title

Important open problems in extremal graph theory.

That lecture and this lecture are strongly connected to each other. Extremal Graph Theory is one of the largest subtheory of Discrete Mathematics and influenced Discrete Mathematics in several ways.

(a) The most important open problems worked as driving forces in Discrete Mathematics. To answer them new methods, subtheories were developed which often turned out very deep theories, with their own questions.

(b) These subtheories led to new conjectures, results and often lead to surprising newer and newer theorem.

(c) Thos conjectures, problems turned out to be important in Mathematics, and therefore in Discrete Mathematics too, which lead to interesting new theories or interesting new tools,

Such cases can be observed in connection with extremal graph theory. Ramsey Theory and Extremal Graph Theory together lead to the Theory of Random Graphs, to many applications of finite geometrical constructions, to Regularity Lemmas, to Graph Limit theory, to the theory of quasi-random structures, in some sense to part of the theory of Property Testing, and to several further subtheories of Discrete Mathematics.

I will discuss such phenomena in my lecture.

## 26 On Hosoya's dormant and sprouts

Dragan Stevanović  
Serbian Academy of Sciences and Arts, Serbia  
dragance106@yahoo.com

The study of cospectral graphs is one of the traditional topics of spectral graph theory. Initial expectation by theoretical chemists Günthard and Primas in 1956 that molecular graphs are characterized by the multiset of eigenvalues of the adjacency matrix was quickly refuted by the existence of numerous examples of cospectral graphs in both chemical and mathematical literature. This work was further motivated by Fisher in 1966 in the influential study that investigated whether one can “hear” the shape of a (discrete) drum, which has led over the years to the construction of many cospectral graphs. These findings culminated in setting the ground for the Godsil-McKay local switching and the Schwenk's use of coalescences, both of which were used to show (around the 1980s) that almost all trees have cospectral mates. Recently, enumerations of cospectral graphs with up to 12 vertices by Haemers and Spence and by Brouwer and Spence have led to the conjecture that, on the contrary, “almost all graphs are likely to be determined by their spectrum”. This conjecture paved the way for myriad of results showing that various special types of graphs are determined by their spectra. On the other hand, in a recent series of papers, Hosoya drew the attention to a particular aspect of constructing cospectral graphs by using coalescences: that cospectral graphs can be constructed by attaching multiple copies of a rooted graph in different ways to subsets of vertices of an underlying graph.

After briefly surveying the history of constructing cospectral graphs, we focus on the expectations and questions about cospectrality of multiple coalescences that were raised in Hosoya's papers. In particular, we discuss the characteristic polynomial of such multiple coalescences, from which a necessary and sufficient condition for their cospectrality follows. We enumerate such pairs of cospectral multiple coalescences for a few families of underlying graphs, and show the infinitude of cospectral multiple coalescences having paths as underlying graphs, which were deemed rare by Hosoya. (This is a joint work with Salem Al-Yakoob.)

## 27 Pseudo-geometric strongly regular graphs with a regular point

Edwin R. van Dam  
Tilburg University, Netherlands  
Edwin.vanDam@tilburguniversity.edu

We study pseudo-geometric strongly regular graphs whose second subconstituent with respect to a vertex is a cover of a strongly regular graph or a complete graph. By studying the structure of such graphs, we characterize all graphs containing such a vertex, and use our characterization to find many new strongly regular graphs. Thereby, we answer a question posed by Gardiner, Godsil, Hensel, and Royle. We give an explicit construction for  $q$  new, pairwise non-isomorphic graphs with the same parameters as the collinearity graph of generalized quadrangles of order  $(q, q)$  and a new non-geometric graph with the same parameters as the collinearity graph of the Hermitian generalized quadrangle of order  $(q^2, q)$ , for prime powers  $q$ .

## 28 On the eccentricity matrix of finite graphs

Jianfeng Wang  
Shandong University of Technology, China  
jfwang@sdut.edu.cn

In eccentricity parlance, the authors standardly defined Professor Randić's  $D_{\max}$ -matrix of a graph and renamed as eccentricity matrix in their paper [J.F. Wang, M. Lu, F. Belardo, M. Randić, The anti-adjacency matrix of a graph: Eccentricity matrix, *Discrete Appl. Math.* 251(2018) 299–309], where they firstly studied its spectral properties. The reasons why the authors tended to build a spectral theory based on the eccentricity matrix were detailedly explained in [J.F. Wang, M. Lu, M. Brunetti, L. Lu, X.Y. Huang, Spectral determinations and eccentricity matrix of graphs, *Adv. Appl. Math.* 139 (2022) 102358]. Since the end of 2018, the eccentricity matrix has been drawing much attention from the researchers more and more. At present, over 20 papers/manuscripts about this topic have been published and submitted for under review. In this lecture, the spectral properties of eccentricity matrix will be surveyed and reported.

## 29 Bijections on Hyperplane Arrangements

Suijie Wang  
Hunan University, China  
wangsuijie@hnu.edu.cn

In this talk, I will introduce some bijections on two classical hyperplane arrangements, the extended Shi arrangement and the extended Catalan arrangement. The most celebrated bijection on hyperplane arrangement is the Pak-Stanley labeling, introduced by I. Pak and extended by R. P. Stanley in the paper “*R. P. Stanley, Hyperplane arrangements, interval orders and trees, Proc. Nat. Acad. Sci. 93 (1996), 2620-2625.*” It establishes a correspondence from regions of the  $k$ -Shi arrangement to  $k$ -parking functions.

In a most recent paper “*R. Duarte, A. Guedes de Oliveira, Pak-Stanley labeling of the  $m$ -Catalan hyper-plane arrangement, Adv. Math. 387 (2021), Paper No. 107827, 17 pp.*” they establish a bijection from fundamental regions of the  $m$ -Catalan arrangement to  $m$ -Dyck paths.

In our paper “*H. Fu, S. Wang, W. Zhu, Bijections on  $r$ -Shi and  $r$ -Catalan arrangements. Adv. in Appl. Math. 129 (2021), Paper No. 102207, 22 pp.*”, we establish a bijection from regions of the  $r$ -Shi arrangement to  $O$ -rooted labeled  $r$ -trees. To this end, we introduce a cubic matrix for each region of the hyperplane arrangements. Our bijection is established by reading the positions of minimal positive entries in its column slices. Moreover, we will recover the

Pak-Stanley labeling by reading the numbers of positive entries in its row slices; and recover the bijection of Duarte and Guedes de Oliveira by reading the numbers of positive entries in its column slice.

## 30 Star edge coloring of 1-planar graphs

Yiqiao Wang

Beijing University of Chinese Medicine, China

yqwang@bucm.edu.cn

The star chromatic index  $\chi'_{\text{st}}(G)$  of a graph  $G$  is defined as the smallest  $k$  for which the edges of  $G$  can be colored using  $k$  colors so that no two adjacent edges get same color and no bichromatic paths or cycles of length four are produced. A graph  $G$  is called 1-planar if it can be drawn in the plane such that each edge crosses at most one other edge. In this talk, we investigate the edge coloring of 1-planar graphs and show that every 1-planar graph  $G$  with maximum degree  $\Delta$  satisfies  $\chi'_{\text{st}}(G) \leq 7.75\Delta + 166$ ; and moreover  $\chi'_{\text{st}}(G) \leq \lfloor 1.5\Delta \rfloor + 500$  if  $G$  contains no 4-cycles, and  $\chi'_{\text{st}}(G) \leq 2.75\Delta + 116$  if  $G$  is 3-connected, or optimal, or NIC-planar.

## 31 Cycles in Color-Critical Graphs

Douglas B. West

Zhejiang Normal University, China

University of Illinois, USA

dwest@illinois.edu

Tuza [1992] proved that a graph  $G$  having no cycles of length congruent to 1 modulo  $k$  is  $k$ -colorable. We strengthen this by proving for  $2 \leq r \leq k$  that any edge  $e$  such that  $G - e$  is  $k$ -colorable and  $G$  is not lies in at least  $\prod_{i=1}^{r-1} (k - i)$  cycles of length  $1 \pmod r$  in  $G$ . Also,  $G - e$  contains at least half that many cycles of length  $0 \pmod r$ .

We also consider an analogue of Tuza's result for circular coloring. A  $(k, d)$ -coloring of a graph  $G$  is a homomorphism from  $G$  to the graph with vertex set  $Z_k$  and edge set  $\{ij : d \leq j - i \leq k - d\}$ . With  $k$  and  $d$  relatively prime, let  $s = d^{-1} \pmod k$ . Zhu [2002] proved that  $G$  has a  $(k, d)$ -coloring when  $G$  has no cycle  $C$  with length congruent to  $is$  modulo  $k$  for any  $i \in \{1, \dots, 2d - 1\}$ . In fact, only  $d$  classes need be excluded: we prove that if  $G - e$  is  $(k, d)$ -colorable and  $G$  is not, then  $e$  lies in at least one cycle with length congruent to  $is \pmod k$  for some  $i$  in  $\{1, \dots, d\}$ .

These results are joint work with Benjamin R. Moore.

## 32 How does an induced Hourglass effect a graph to have Hamiltonian properties?

Liming Xiong

Beijing Institute of Technology, China

lmxiong@bit.edu.cn

In this talk, we pay our attention on the induced Hourglass (a connected graph with degree sequence 4,2,2,2,2). We list results to show how to effect a graph to have Hamiltonian properties (Hamiltonicity or Hamilton-connectivity, et.al.), when we add the hourglass-free condition on 3-connected graphs.

### 33 Counting critical subgraphs in $k$ -critical graphs

Tianchi Yang

University of Science and Technology of China, China

ytc@mail.ustc.edu.cn

Gallai asked in 1984 if any  $k$ -critical graph on  $n$  vertices contains at least  $n$  distinct  $(k-1)$ -critical subgraphs. The answer is trivial for  $k \leq 3$ . Improving a result of Stiebitz, Abbott and Zhou proved in 1995 that for all  $k \geq 4$ , any  $k$ -critical graph contains  $\Omega(n^{1/(k-1)})$  distinct  $(k-1)$ -critical subgraphs. Since then no progress had been made until very recently, Hare resolved the case  $k = 4$  by showing that any 4-critical graph on  $n$  vertices contains at least  $(8n - 29)/3$  odd cycles.

In this talk, we mainly focus on 4-critical graphs and develop some novel tools for counting cycles of specified parity. Our main result shows that any 4-critical graph on  $n$  vertices contains  $\Omega(n^2)$  odd cycles, which is tight up to a constant factor by infinitely many graphs. As a crucial step, we prove the same bound for 3-connected non-bipartite graphs, which may be of independent interest. Using the tools, we also give a short solution to Gallai's problem when  $k = 4$ . Moreover, we improve the longstanding lower bound of Abbott and Zhou to  $\Omega(n^{1/(k-2)})$  for the general case  $k \geq 5$ . We will also discuss related problems on  $k$ -critical graphs. This is joint with Prof. Jie Ma.

### 34 Bollobás-type theorems for hemi-bundled two families

Xiande Zhang

University of Science and Technology of China, China

drzhangx@ustc.edu.cn

In this talk, we discuss a Bollobás-type theorem on the number of cross-intersecting set pairs  $(A_i, B_i)$ , with additional condition that all  $A_i$  sets are intersecting. The theorem is proved by using exterior algebra method.

### 35 New Routing Problem in FPGA Chip Design

Xiaoyan Zhang

Nanjing Normal University, China

05373@njnu.edu.cn

The IC has developed rapidly as the birth of the first transistor in the world. The characteristics of small size, large density and high integration of modern chips make routing design more and more difficult. In the system level FPGA routing design problem, each edge has different time TDM ratios in different FPGA nets when using time division multiplexing technology, and unreasonable TDM ratio will lead to increasing signal delay of the whole system. In order to reduce the signal delay of the whole system, it is important to design a reasonable routing scheme considering the TDM ratio. For the task, we first propose a method of constructing approximate minimum Steiner tree based on shortest path and disjoint shortest path respectively, which is applied to find the specific routing path of signal transmission in dual FPGA net and multi-FPGA net respectively. Then, a time-sharing multiplexing ratio allocation scheme based on group sizes of FPGA networks is proposed to allocate the TDM ratio for each routing path of FPGA network.

## 36 Some properties from real-rooted polynomials in combinatorics

Baoxuan Zhu

Jiangsu Normal University, China

bxzhu@jsnu.edu.cn

The zeros of polynomials play an important role in mathematics. In this talk, we will introduce some properties including unimodality, log-concavity,  $q$ -log-convexity, total positivity,  $\gamma$ -positivity, continued fractions, and so on, from real rootedness of combinatorial polynomials.



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